ON HEAT TRANSFER IN MHD CHANNEL FLOW UNDER CROSSED-FIELDS

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ABSTRACT. An investigation of the combined influence of a magnetic field and an electric field on convective heat transfer in the fully developed laminar flow of an incompressible conducting fluid between parallel plates has been carried out, thus extending an earlier analysis of Siegal.

An analysis of heat transfer in a mhd channel flow under uniform magnetic field and heat flux at the non-conducting walls was presented by Siegel (1958) and that at the conducting walls was presented by Alpher (1961) and Yen (1963). In engineering applications, the mhd channel flows have been characterised by Sutton and Sherman (1965) with the help of loading parameter K $(= E_y/B\bar{u}_0)$, where E_y is the applied electric field. It is a generator, accelerator or flowmeter according as $K >, <$ or $= 0$ respectively. Recently Soundalgekar (1968) discussed the heat transfer aspect of fully developed mhd channel flow between conducting walls under crossed-fields. In this note an analysis of the effects of K, M and the uniform heat flux on the heat transfer aspect of the mhd channel flow between non-conducting walls is presented.

The expressions for the velocity profile and the current density in non-dimensional form as derived by Sherman and Sutton (1965) are

$$u = \frac{M\,(\text{Cosh } M - \text{Cosh } MZ)}{M \text{ Cosh } M - \text{Sinh } M} \quad .. \quad (1)$$

and

$$J = K - u \quad .. \quad (2)$$

The energy equation neglecting viscous dissipation is

$$u\,\frac{\partial T}{\partial x} = \alpha\,\frac{\partial^2 T}{\partial z^2} + \frac{j_y^2}{\rho c_p} \quad .. \quad (3)$$

where $\alpha = \lambda/\rho c_p$ is the thermal diffusivity.

For linearly varying temperature along the plates, one can assume

$$T(x, z) = Ax + G(z) \quad .. \quad (4)$$

where A is the mean temperature gradient. In view of (2) and (4), equation (3) reduces to

$$\frac{d^2G}{dZ^2} = \frac{A\bar{u}L^2}{\alpha} u - \frac{M^2P_r\bar{u}^2}{c_p} (K-u)^2, \quad .. \quad (5)$$

where P , $\bar{u}$, c , L are the Prandtl number, mean velocity, specific heat and half channel width respectively.

The boundary conditions are (Siegel 1958)

$$G = 0 \text{ at } Z = \pm 1 \text{ and } \frac{dG}{dZ} = 0 \text{ at } Z = 0. \quad .. \quad (6)$$

The solution of (5) subject to (6) is

$$G = C_1 + \frac{\bar{u}AL^2M}{\alpha B}\left[\frac{Z^2\cosh M}{2} - \frac{\cosh MZ}{M^2}\right] -$$

$$- \frac{P_rM^2\bar{u}^2}{c_p}\left[\frac{K^2Z^2}{2} - \frac{2KM}{B}\left(\frac{Z^2\cosh M}{2} - \frac{\cosh MZ}{M^2}\right) +\right.$$

$$\left. + \frac{M^2}{B^2}\left\{\frac{Z^2(2+\cosh 2M)}{4} - \frac{2\cosh M\cosh MZ}{M^2} + \frac{\cosh 2MZ}{8M^2}\right\}\right], \quad .. \quad (7)$$

where

$$C_1 = \frac{P_rM^2\bar{u}^2}{c_p}\left[\frac{K^2}{2} - \frac{K(M^2-2)\cosh M}{BM} +\right.$$

$$\left. + \frac{4(M^2-2)+(2M^2-7)\cosh 2M}{8B^2}\right] - \frac{\bar{u}^2AL^2}{\alpha B}\frac{(M^2-2)\cosh M}{2M},$$

$$B = M\cosh M - \sinh M.$$

The mean temperature gradient A is found by considering the overall heat balance for a differential length of the channel, which gives

$$A = \frac{\partial T}{\partial x} = \frac{\partial T_m}{\partial x} = \frac{q + \int_0^L \frac{j_y^2}{\sigma}\, dz}{L\bar{u}\rho c_p} \quad .. \quad (8)$$

From (1), (2) and (8), we get after integration

$$A = (L\bar{u}\rho C_p)^{-1}\left[q + \frac{M^2\bar{u}\mu}{L}\left\{\frac{M^2}{2B^2} - K +\right.\right.$$

$$\left.\left. + \frac{M\cosh M(2M\cosh M - 3\sinh M)}{B^2}\right\}\right] \quad .. \quad (9)$$

When the mean temperature of the fluid is known, it is of practical importance to determine the wall temperature. Hence we form the difference between wall and mean fluid temperature as

$$T_w - T_m = AX - \frac{1}{2L\bar{u}} \int_{-L}^{L} T(x, z)\, u(z) dz$$

$$= -\tfrac{1}{2} \int_{-1}^{1} G(Z)\, u(Z) dZ. \qquad \ldots \quad (10)$$

Substituting for G, A and u from (7), (9) and (1) respectively in (10), integrating and rearranging, we have

$$T_w - T_m = \frac{qL}{\lambda} \Phi(M) + \frac{P_r{}^2 R^2 \nu \alpha}{L^2 c_p} \psi(M, K), \qquad (11)$$

where

$$\Phi(M) = \frac{(M^2-2)\cosh M}{2BM} - \frac{3M^2 - 9\cosh M \sinh M + M(M^2+1)\cosh^2 M}{6MB^2} \qquad \ldots \quad (12)$$

$$\psi(M,K) = \frac{M}{B}\left(\frac{M^2}{2B^2} - K + \frac{2M\cosh^2 M\,(M - \tanh M)}{B^2}\right).$$

$$\cdot\left(\frac{(M^2-2)\cosh M}{2} - \frac{3M^2 - 9\cosh M \sinh M + M(M^2+6)\cosh^2 M)}{6B}\right) +$$

$$+ \frac{K^2 \cosh M}{6B}\left(M^3 - 3\tanh M.(M^2+2) + 6M\right) -$$

$$-\frac{KM^2}{3B^2}\left(3M^2 - 9\cosh M \sinh M + M(M^2+6)\cosh^2 M\right) +$$

$$+ \frac{M^3 \cosh M}{3B^2}\left(\frac{M^2+2}{3} - \frac{(33+8M^2)\tanh M}{8M} + \right.$$

$$+ \frac{M^2+6}{6}\cosh 2M - \frac{M^2+6}{2M}\tanh M \cosh 2M -$$

$$\left. - \frac{9\sinh 2M}{8M} - \frac{\sinh 3M}{24M\cosh M}\right). \qquad \ldots \quad (13)$$

To obtain the mean temperature at a particular x-position, measured from the channel entrance, we integrate equation (10) and obtain

$$T_m - T_0 = \frac{x}{P_r R}\left\{\frac{q}{\lambda} + \frac{(P_r R)^2 \nu \alpha}{L^3}\left[\frac{M^2}{2B^2} - K + \right.\right.$$

$$\left.\left. + \frac{M\cosh^2 M(2M - 3\tanh M)}{B^2}\right]\right\}, \qquad \ldots \quad (14)$$

where T_0 is the mean temperature of the fluid entering the channel.

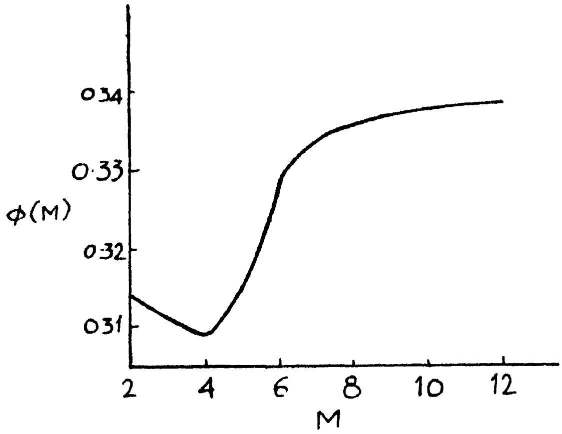

Figure 1. $\phi(M)$ vs Hartmann number M.

In order to know the variation of $T_w - T_m$, the functions $\Phi(M)$ and $\psi(M, K)$ are calculated for $M = 2, 4, 6, 8, 10, 12$ and $K = 0, 0{\cdot}2, 0{\cdot}4, 0{\cdot}6, 0{\cdot}8, 1$. $\Phi(M)$ is shown in figure 1, whereas the numerical values of $\psi(M,K)$ are entered in table I.

Table 1 Values of $\psi(M, K)$

K \ M	2	4	6
0	−25	92	3861
0.2	−28	82	3835
0.4	−31	72	3810
0.6	−34	62	3785
0.8	−37	52	3761
1.0	−41	42	3737

CONCLUSIONS

An increase in K leads to a decrease in $\psi(M, K)$ whereas it increases with M.

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